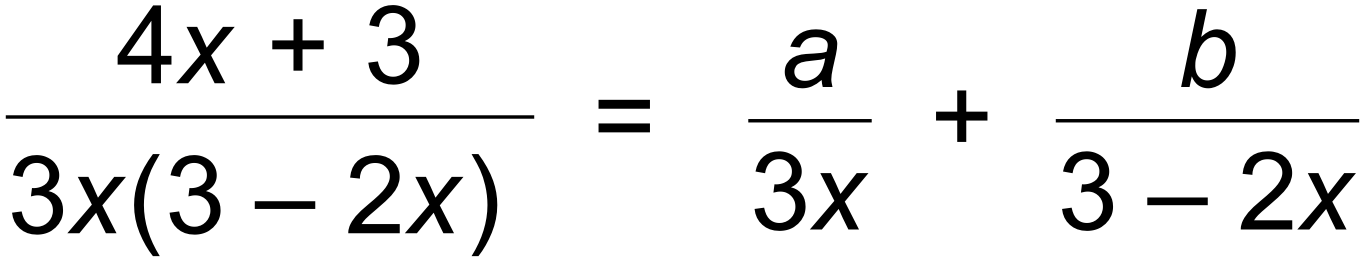
**Mathematics Specialist Test 4 2018 Calculator Free Marking Key**

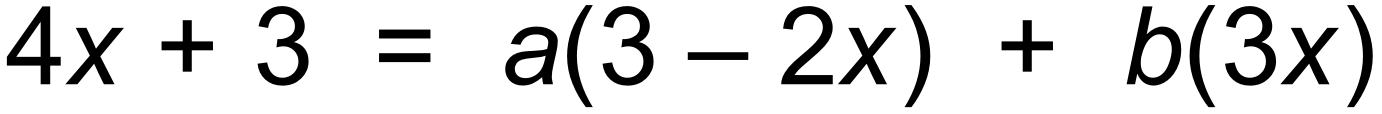
**Question 1(a)**

|  |
| --- |
| **Solution** |
| Then |
| **Specific behaviours** |
| 🗸 changes variable to *u* in integral  🗸 anti-differentiates with respect to *u*  🗸 evaluates correctly |

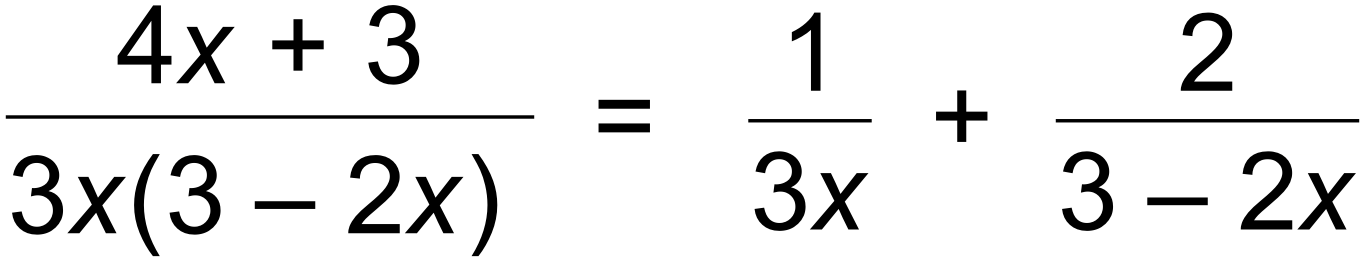
**Question 1 (b)**

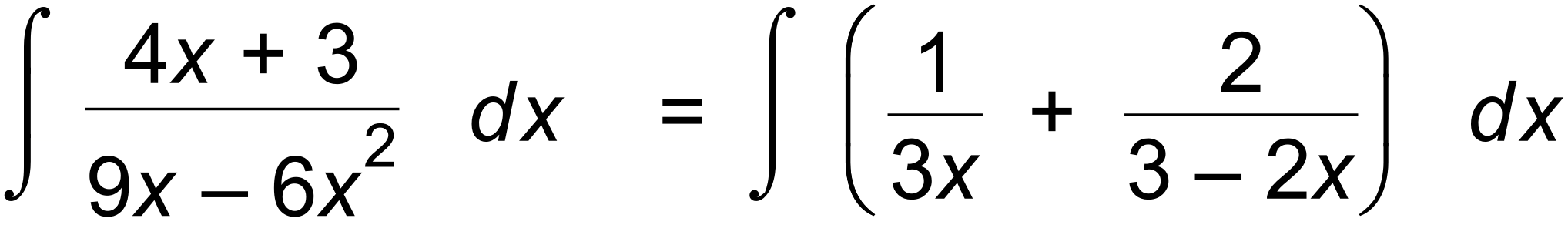
|  |
| --- |
| **Solution** |
| Then    Hence    Thus the integral equals 1 if |
| **Specific behaviours** |
| 🗸 changes variable to *v* in integral  🗸 evaluates the integral correctly  🗸 deduces the correct value of *Q* |

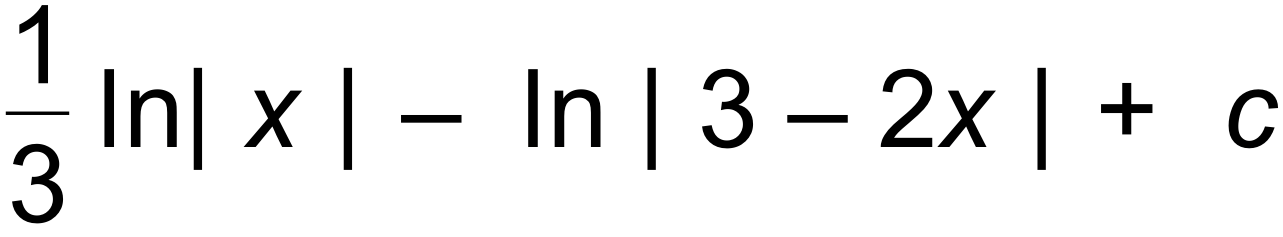
2. (a)  ✓

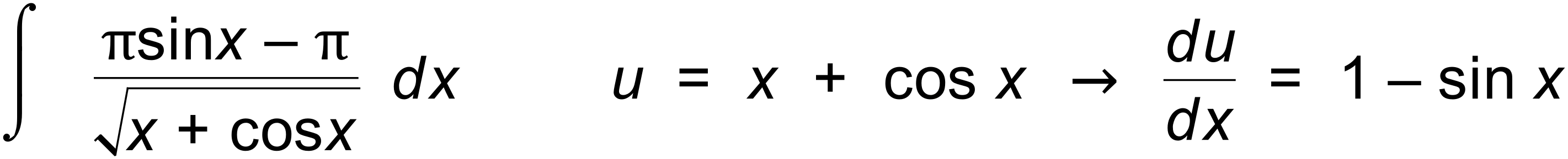
∴ 

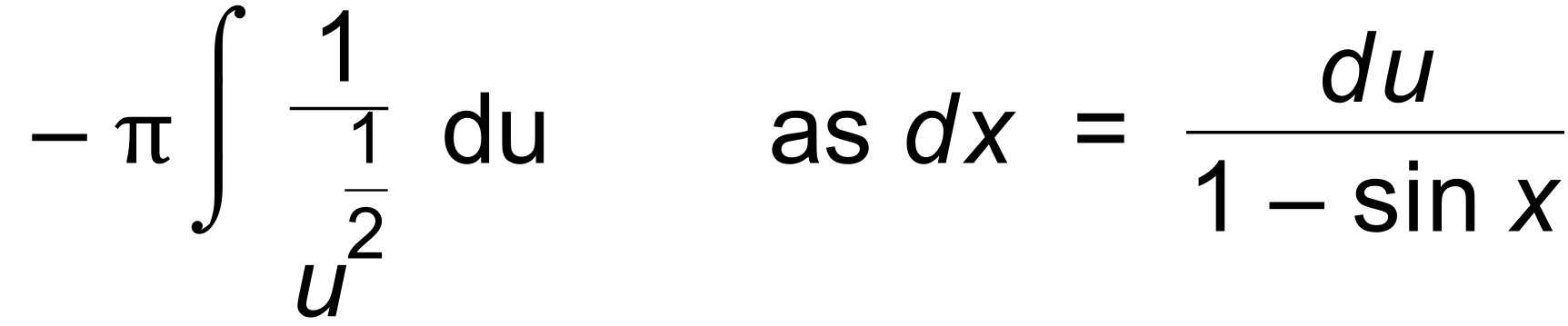
∴  ✓

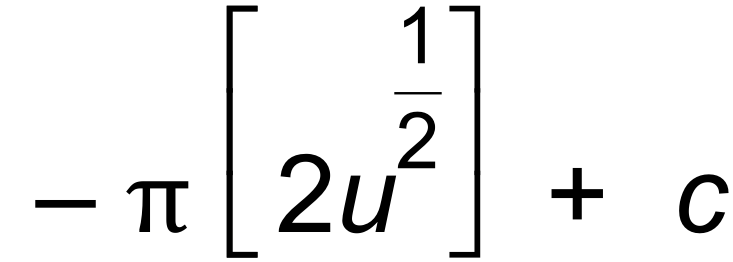
∴  ✓

(b)  ✓

=  ✓✓ [6]

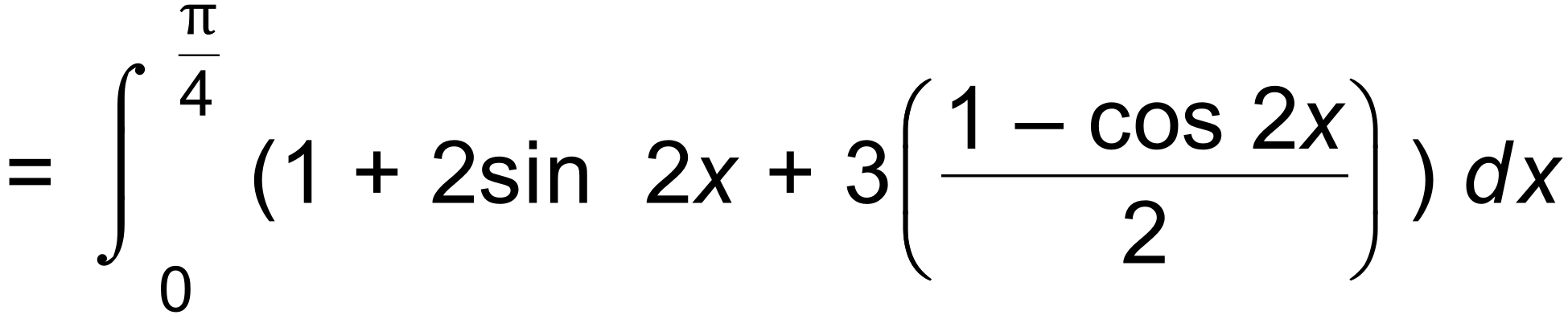
3. (a)  ✓

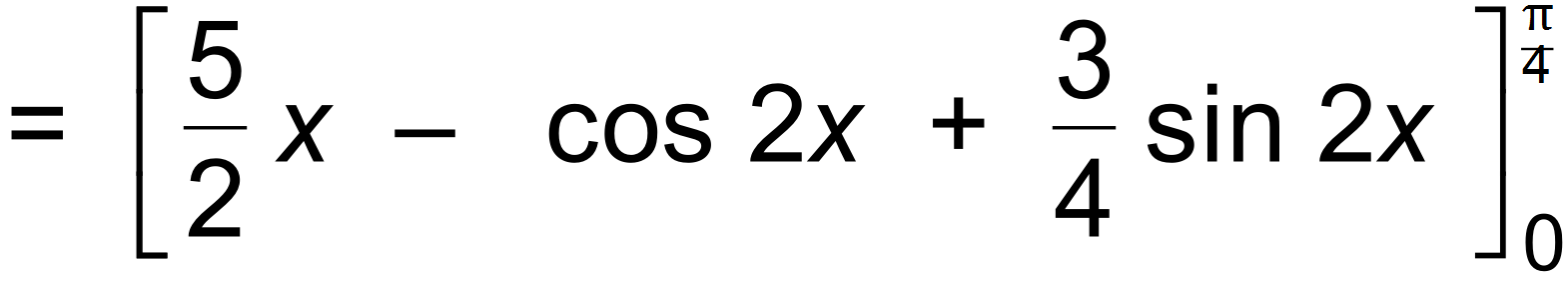
=  ✓

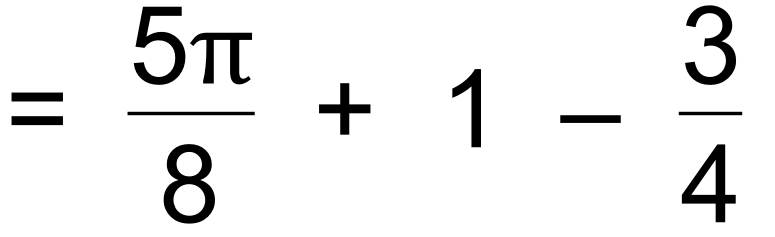
=  ✓

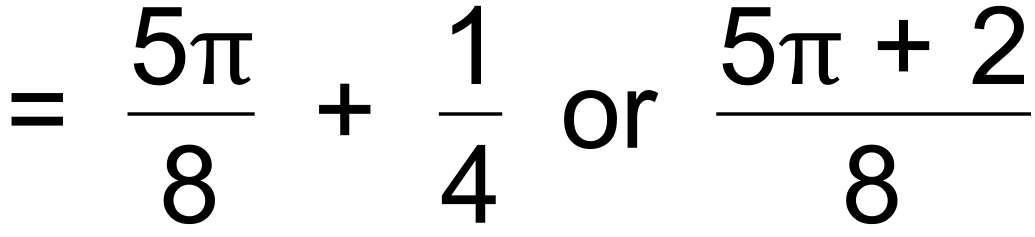
=  ✓

(b)  ✓

 ✓

 ✓



 ✓ [8]

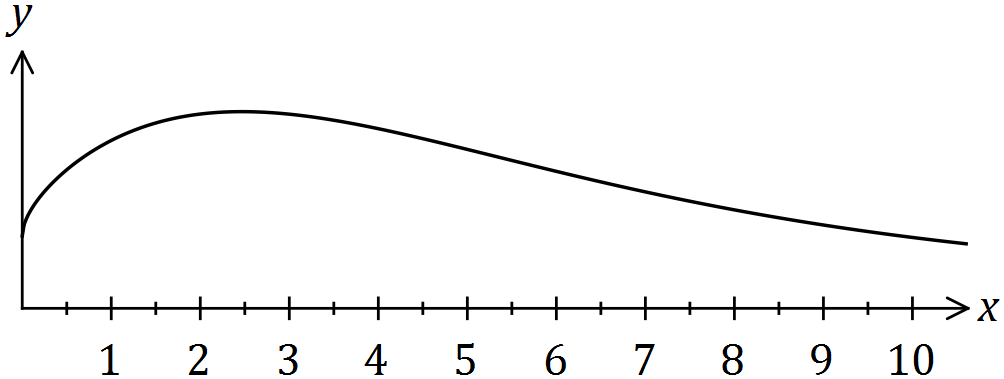
4. ✓

= ✓

= = ✓✓**MATHEMATICS SPECIALIST TEST 4 2018 MARKING KEY CALCULATOR ASSUMED**

Question 5 (5 marks)

Part of the graph of is shown below.



(a) Use numerical integration with three equal width trapeziums to estimate the area between the curve, the -axis, the -axis and the line . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates for  ✓ uses trapezium rule  ✓ area of one trapezium correct to at least 3rd decimal place  ✓ area rounded to 3 or more (ie 4, 5, ...) sf |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates for  ✓ uses trapezium rule  ✓ area of one trapezium correct to at least 3rd decimal place  ✓ area rounded to 3 or more (ie 4, 5, ...) sf |

(b) Briefly explain how your estimate in (a) would change if thirty trapeziums were used.

(1 mark)

|  |
| --- |
| **Solution** |
| Estimate would increase. |
| **Specific behaviours** |
| ✓ indicates increase |

Question 6 (8 marks)

The velocity vector of a small body at time seconds is ms-1. Initially, the body has position vector .

(a) Determine the acceleration vector for the body when . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ acceleration vector  ✓ acceleration |

(b) Show that the maximum speed of the body is ms-1. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for or  ✓ simplifies expression  ✓ indicates maximum with justification |

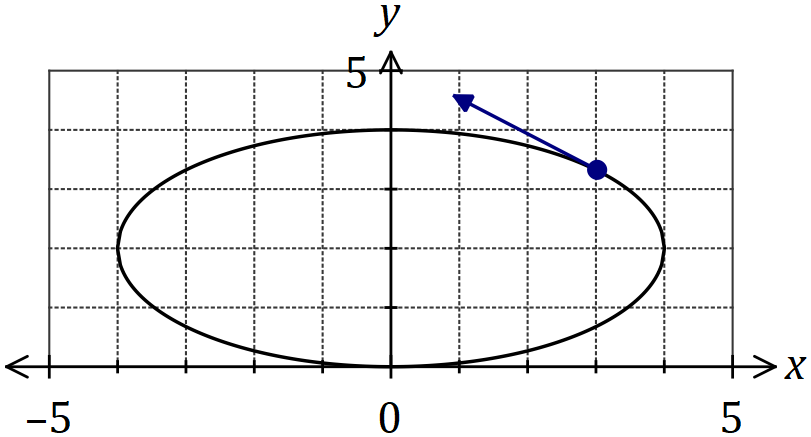
(c) Determine the distance the body travels between and the first instant after this time that the body returns to its initial position, rounding your answer to the nearest cm.

(3 marks)

|  |
| --- |
| **Solution** |
| Period of motion is |
| **Specific behaviours** |
| ✓ period  ✓ indicates valid integral  ✓ distance (*no rounding penalty*) |

Question 7 (8 marks)

The position vector of a boat motoring on a lake is given by , where is the time, in hours, after it leaves and distances are in kilometres. The path of the boat is shown below, where the shoreline is represented by the line .



|  |
| --- |
| **Solution (b)** |
| See graph |
| **Specific behaviours** |
| ✓ position and direction |

(a) Express the path of the particle as a Cartesian equation. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses double angle identity  ✓ isolates and  ✓ Cartesian equation |

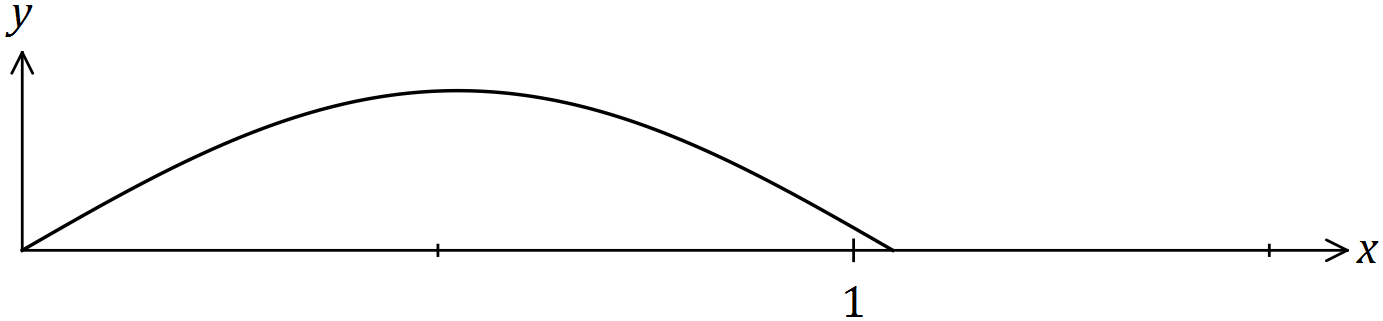
(b) On the graph above, mark the position of the boat when it is first km from the shoreline and indicate the direction it is travelling. (1 mark)

(c) Determine the speed of the boat when it is first km from the shoreline. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines time  ✓ obtains velocity vector  ✓ velocity vector at time  ✓ speed |

Question 8 (5 marks)

Part of the graph of is shown below.



Show that when the part of the curve between and is rotated about the axis, the volume of the solid generated is . Clearly indicate all trigonometric identities used.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct integral to evaluate  ✓ shows identity for  ✓ substitutes to obtain expression in terms of  ✓ integrates correctly  ✓ substitutes all limits (and simplifies) |